Grade 11—Mathematics

The **academic expectation** addressed by "Determining a Square" is

2.9   Students understand space and dimensionality concepts and use them appropriately and accurately.

The **core content** assessed by this item includes

Geometry/Measurement Concepts

- Students should understand spatial relationships such as betweenness, perpendicularity, and parallelism.

Geometry/Measurement Relationships

- Students should understand how properties of geometric shapes relate to each other.

Determining A Square

Jamie, Chris, and Pat are outlining a square foundation for a storage building. They have string, tape measures, and a protractor. Each person's method for forming a square is given below:

Jamie's method: "Cut four strings that have the same length as the sides of the square storage building. Place these strings to form a quadrilateral. That quadrilateral will be a square."

a.   Will Jamie's method **always** form a square? Justify your reasoning using the properties of squares.

Chris' Method: "Cut four strings that have the same length as the sides of the square storage building. Place these strings to form a quadrilateral, making sure that two of the adjacent sides form a right angle."

b.   Will Chris' method **always** form a square? Justify your reasoning using the properties of squares.

Pat's Method: "Cut two strings the same length as diagonals of the square base of the storage building. Fold them in half, marking the center of each string. Unfold the strings and place them on the ground so that they intersect at their centers to form an x. Connect the endpoints of the strings to form a quadrilateral. That quadrilateral will be a square."

c.   Will Pat's method **always** form a square? Justify your reasoning using the properties of squares.

BE SURE TO LABEL YOUR RESPONSES (a), (b), and (c).

Score	Description
4	Student scores 6 points.
3	Student scores 4–5 points.
2	Student scores 2–3 points.
1	Student scores 1 point. **OR** Student scores 0 points, but provides at least one correct answer. **OR** Student identifies some characteristics of a square that could be used to prove or disprove arguments.
0	Response is totally incorrect or irrelevant.
Blank	No response.

Answers

Part a   No. Jamie forms a four-sided figure but equal or right angles cannot be guaranteed.
OR
Student gives a counterexample.

Part b   Yes. Chris' quadrilateral has the adjacent angles equal and therefore all the angles equal, and right angles. The figure will be a square. (Student may prove by pointing out that since it must be a rhombus, with one right angle, it must be a square.)

Part c   No. Pat's figure has diagonals that are equal and bisect each other. Therefore, the figure determined is a rectangle. The diagonals would also need to be perpendicular in order to determine a square. (Student may use an equivalent drawing that shows a counterexample: non-perpendicular diagonals which yield a rectangle that is not a square.)

Score Points

Part a   Score 2 points   Correct answer with complete explanation
OR

Score 1 point   Correct answer with incomplete explanation
OR

Score 1 point   Correct answer with complete explanation that contains some incorrect information
OR

Score 1 point   Student answers "Yes," and provides complete explanation why Jamie's method would <u>not</u> work.

OR

Score 0 points   Correct answer with incorrect or missing explanation
OR

Score 0 points   Incorrect answer

Part b   Score 2 points   Correct answer with complete explanation
OR

Score 1 point   Correct answer with incomplete but not incorrect explanation
OR

Score 1 point   Correct answer with complete explanation that contains some incorrect information
OR

Score 1 point   Student answers "No," and provides complete explanation why Chris's method <u>would</u> work

OR

Score 0 points   Correct answer with incorrect or missing explanation
OR

Score 0 points   Incorrect answer

Part c   Score 2 points   Correct answer with complete explanation
OR

Score 1 point   Correct answer with incomplete but not incorrect explanation
OR

Score 1 point   Correct answer with complete explanation that contains some incorrect information
OR

Score 1 point   Student answers "Yes," and provides complete explanation why Pat's method would <u>not</u> work.

OR

Score 0 points   Correct answer with incorrect or missing explanation
OR

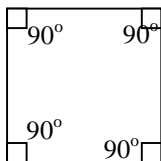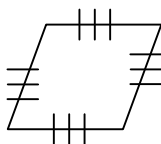Score 0 points   Incorrect answer

Sample 4-Point Response of Student Work

Student Response

a) Jamie's method won't always be a square. One of the properties of a square is that all angles = 90°.

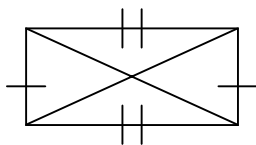If Jamie had 4 strings the same size and arranged them like this, it wouldn't be a square.

Student provides correct answer with complete explanation for part a (2 points).

b) Chris's method would always form a square. This is because since the strings are the same size and one angle is 90°, all the angles have to be 90°. That is a property of a square.

Student provides correct answer with complete explanation for part b (2 points).
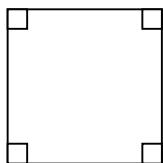
c) Pat's method won't always work. Although, Pat would always establish four 90° angles, all 4 sides aren't equal. One property of a square is that all sides are equal.

Student provides correct answer with complete explanation for part c (2 points).

Overall, the student earns 6 points and demonstrates a solid understanding of spatial relationships and geometric properties by fully responding to the prompt with complete and correct explanations. The student clearly states in each part the property of a square that is met and the property that is not met. The diagrams are clear and helpful.

Sample 4-Point Response of Student Work

Student Response

a) No, Jamie's method will not always work. A square has four congruent sides and 4 congruent angles. She would not be able to form a 90° angle every time.

b) Yes, Chris's method will work every time. A quadrilateral with 4 congruent sides with even one proven right angle will have three other right angles forming a square.

c) No, Pat's method will not work. In order for a quadrilateral to have congruent diagonals bisecting each other, the quadrilateral must be a rectangle. In order for it to be a square the diagonals must bisect at a right angle.

Student provides correct answer with complete explanation for part a (2 points).

Student provides correct answer with complete explanation for part b (2 points).

Student provides correct answer with complete explanation for part c (2 points).

Overall, the student earns 6 points and demonstrates a solid understanding of spatial relationships and geometric properties by fully responding to the prompt with complete and correct explanations. The student clearly states in each part the property of a square that is met and the property that is not met.
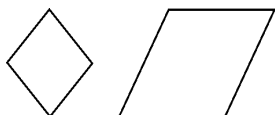
Sample 3-Point Response of Student Work

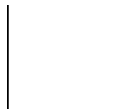## Student Response

a. Four equal lines in length or in this case, strings formed as a quadrilateral will not always be a square. It could be a rhombus or a parallelogram:
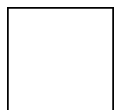
Neither of these are squares.

b. Lines with the same length, formed as a quadrilateral, with two adjacent sides forming a right angle will always form a square. I'll start with the right angle:
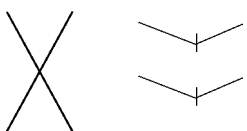
The two other strings must connect to form a quadrilateral. So I have no choice but to make them a right angle as well.
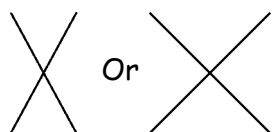
A quad. with two adjacent sides forming a right angle (and all sides being equal) always form a square.
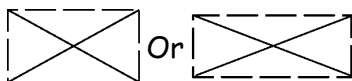
c. Pat took two diagonals and folded them in half:

Placing them on the ground, she says to unfold them and let them intersect at the mark, and it will be a square: (for example)

Or

The lines can rotate almost 180° each, and still match up to the mark:

Or

definitely does not always make a square, because the length and width will not always be equal.

Student provides correct answer with incomplete explanation for part a (1 point).

Student provides correct answer with complete explanation for part b (2 points).

Student provides correct answer with complete explanation for part c (2 points).

Overall, the student earns 5 points and demonstrates an understanding of spatial relationships and geometric properties by responding correctly to most of the prompt. **Note:** The student made good use of diagrams (counterexamples). However, the student made a common mistake by implying in *part a* that a square, parallelogram and rhombus are three completely different shapes, thus making an incorrect statement.
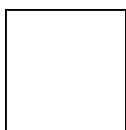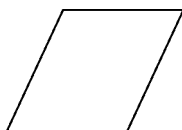
Sample 2-Point Response of Student Work

Student Response

a. Jamie's method will not always form a square, because a quadrilateral with equal sides can also be a rhombus

Square   rhombus

b. Chris' method will always form a square, because a square is the only quadrilateral with equal sides and right angles (90°).

c. Pat's method, will not always form a square because the angles of the strings must be equivalent, otherwise they can form other quadrilaterals, such as a rectangle.

Student provides correct answer with incomplete explanation for part a (1 point).

Student provides correct answer for part b, but explanation is lacking (0 points).

Student provides correct answer with incomplete explanation for part c (1 point).

Overall, the student earns 2 points and demonstrates gaps in understanding of spatial relationships and geometric properties by responding correctly to only part of the prompt.
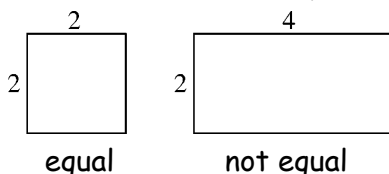
Sample 1-Point Response of Student Work
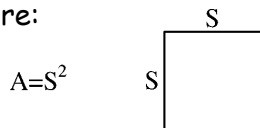
Student Response

a.   Jamie, Chris, and Pat have decided to outline a square foundation for a storage building. Their materials are strings, tape measures, and a protractor. They all three have different methods for creating a foundation.

Jamie's method is to cut 4 strings that have the same length as the sides of the square storage building. Then she will place the squares to form a quadrilateral.
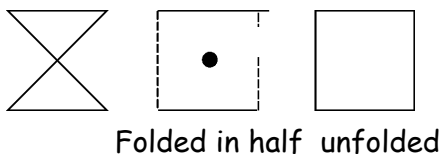
Every time Jamie cuts the string, it will not always form a square. She could make the length longer than the width, however, she will always form a quadrilateral. Here is picture to show this:

```
   2              4
 ┌──────┐   ┌──────────────┐
2│      │  2│              │
 │      │   │              │
 └──────┘   └──────────────┘
  equal        not equal
```

> Student provides a correct answer with an incorrect explanation (i.e., that the length and width would be unequal) for part a (0 points).

b) Chris's method will always form a square. He says to cut 4 strings that have the same length as the sides of the square building. He makes sure that the two adjacent sides form a right angle. Here is a picture:

$$A=S^2$$

```
        S
    ┌──────┐
  S │      │
    │      │
    └──────┘
```

> Student provides a correct answer for part b without explanation (i.e., restates the problem but does not explain that forming one right angle **causes** the other angles to be right angles) (0 points).

c) Pat's method is to cut two strings the same length as diagonals of the square base of the storage building. Then he folds them in half to make the center of each string. Next, he unfolds the strings and places them on the ground so they intersect at their centers to form an X. He finally connects the strings endpoints to form a quadrilateral. It will then be a square. Here is a picture to show this:

> Student provides an incorrect answer for part c (0 points).

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  ╲   ╱   ┌ ─ ┐   ┌──────┐
   ╲ ╱    │ • │   │      │
   ╱ ╲    └ ─ ┘   └──────┘
  ╱   ╲
Folded in half  unfolded
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> Overall, the student earns 0 points. However, the response is not totally incorrect or irrelevant and therefore receives a score of 1. The student demonstrates limited understanding of spatial relationships and geometric properties by responding incorrectly to most of the prompt.

The open-response item **"Determining a Square"** is designed to assess students' (1) understanding of properties of geometric shapes (2) understanding of relationships between geometric shapes including common and distinct properties, and (3) understanding the specific properties of parallelograms. The instructional strategies below present ideas for helping students practice and master these concepts and skills.

Review the properties of polygons. Use a variety of methods for students to explore and demonstrate understanding of how geometric properties relate to each other.

Review the need for precision in statements of fact. For example, the statement "the shape could be a square or a rhombus" implies that a square is not a rhombus. Therefore this statement is considered an incorrect statement. Have students produce imprecise statements and then rewrite them to be more precise.

Review the concept of counterexample. Remind students that one counterexample proves an "always" statement false.

Review the use and importance of clear diagrams to illustrate and clarify statements.

Teach students a variety of strategies for "proving" a conclusion. Note the importance of stating what is evident from the given facts, then proceeding logically from those facts to other facts, and explaining clearly and completely the logical progression (i.e., how **a** leads to **b**, how **b** leads to **c**, etc.).

Provide opportunities for students to work individually, in pairs, in small groups, and/or as a class to complete (with teacher support and guidance) any or all of the following activities:

- Explore, learn, and review the properties of geometric shapes.
- Determine both the common and distinct properties of various shapes.
- Practice using correctly marked diagrams to clearly illustrate statements.
- Practice using counterexamples to disprove conjectures.
- Write a variety of clear and complete proofs.